Quantum friction and non-equilibrium fluctuation theorems

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Work done in collaboration with
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Outline of this Talk

- (Some) previous quantum friction calculations
- Atom-surface interaction: equilibrium
  - Fluctuation-dissipation vs quantum regression
- Atom-surface interaction: non-equilibrium
  - Fluctuation-dissipation vs quantum regression
  - Moving oscillator
  - Moving two-level atom

A variety of predictions

- Mahanty 1980 $$F=-\frac{\hbar\alpha(0)}{32z_a^5}\frac{\epsilon(0)-1}{\epsilon(0)+1}v_x$$
- Schlaich & Harris 1981 $$F=-\frac{\alpha^2(0)e^4}{\hbar\omega_s^2 z_a^{10}}v_x$$
- Tommassone & Widom 1997 (electric dipole + FDT)

$$F=-v_x\frac{3\hbar}{2\pi z_a^5}\int_0^\infty d\omega\frac{\partial n(\omega)}{\partial\omega}\Delta_I(\omega)\alpha_I(\omega)\quad\to 0 \text{ for } T=0 \qquad \Delta(\omega)=\frac{\epsilon(\omega)-1}{\epsilon(\omega)+1}$$

- Volokitin & Persson 2002 Same result (electric dipole + Lorentz force)
- Dedkov & Kyasov 2002-... $dW/dt=-Fv_x$

$$F=\frac{2\hbar}{\pi^2}\int_0^\infty dk_x k_x\int_{-\infty}^\infty dk_y k e^{-2kz_a}\int_0^{k_xv_x}d\omega\Delta_I(\omega)\alpha_I(\omega-k_xv_x)\ \propto v_x^3$$

- Scheel & Buhmann 2009 two- (multi-) level atom + master equation + QRT

$$F=-v_x\frac{d^2\Omega\gamma_a}{2z_a^5}\int_0^\infty d\xi\frac{\Omega^2-3\xi^2}{(\Omega^2+\xi^2)^3}\Delta(i\xi)$$

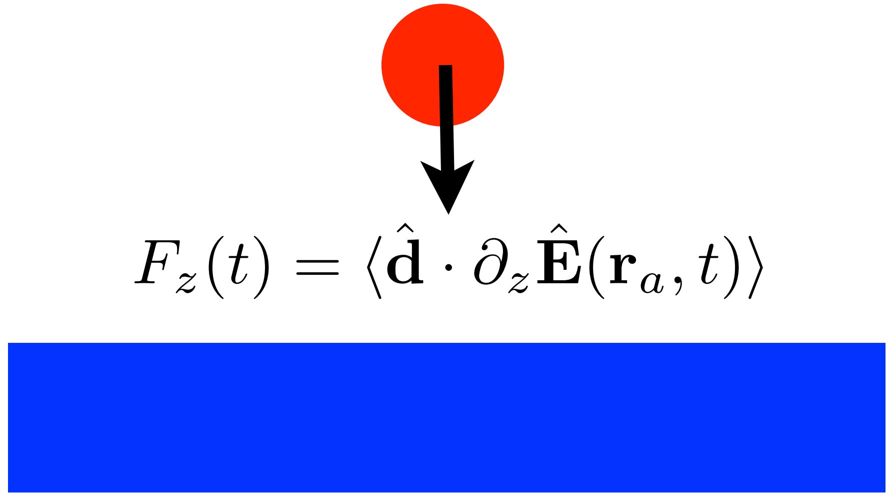
- Barton 2010 Same result SB
- Kardar et al 2013 Same result as TW+VP+DK

Equilibrium case

- Zero temperature $T = 0$
- Uncorrelated initial atom+field/matter

$$\hat{\rho}(0) = \hat{\rho}_a(0) \otimes \hat{\rho}_{\rm fm}(0)$$

- Ground state atom + vacuum field/matter
- Electric field operator



$$F_z(t) = \langle \hat{\mathbf{d}} \cdot \partial_z \hat{\mathbf{E}}(\mathbf{r}_a, t) \rangle$$

$$\hat{\mathbf{E}}(\mathbf{r}, t) = \hat{\mathbf{E}}_0^{(+)}(\mathbf{r}, t) + \frac{i}{\hbar} \int_0^\infty d\omega \int_0^t d\tau e^{-i\omega\tau} \underline{G}_I(\mathbf{r}, \mathbf{r}_a, \omega) \cdot \hat{\mathbf{d}}(t - \tau) + h.c.$$

- Normal force on the atom

$$F_z(t) = \mathrm{Re}\left\{ \frac{2i}{\pi} \int_0^\infty d\omega \int_0^t d\tau e^{-i\omega\tau} \mathrm{Tr}\left[\langle \hat{\mathbf{d}}(t) \hat{\mathbf{d}}(t-\tau) \rangle \cdot \partial_z \underline{G}_I(\mathbf{r}_a, \mathbf{r}_a, \omega) \right] \right\}$$

$$\underline{C}_{ij}(t, t-\tau) \equiv \langle \hat{d}_i(t) \hat{d}_j(t-\tau) \rangle$$

CP: Fluctuation-dissipation

- Stationary $(t \to \infty)$
  density matrix of coupled system

$$\hat{\rho}(\infty) = \hat{\rho}_{\mathrm{KMS}} \propto e^{-\beta \hat{H}}$$

(Kubo-Martin-Schwinger)

- Large time correlator $\underline{C}_{ij}(\tau) = \mathrm{tr}\left\{\hat{d}_i(0)\hat{d}_j(-\tau)\hat{\rho}_{\mathrm{KMS}}\right\}$

- Fluctuation-dissipation (FDT)

power spectrum

$$\boxed{\underline{S}(\omega) = \frac{\hbar}{\pi}\theta(\omega)\underline{\alpha}_I(\omega)}$$

polarizability

$$\underline{S}(\omega) = (2\pi)^{-1}\int_{-\infty}^{\infty} d\tau e^{i\omega\tau}\underline{C}(\tau)$$

$$\underline{\alpha}(\tau) = (i/\hbar)\theta(\tau)\mathrm{tr}\{[\hat{\mathbf{d}}(0), \hat{\mathbf{d}}(-\tau)\hat{\rho}_{\mathrm{KMS}}\}$$

- Stationary Casimir-Polder force

$$\boxed{F_{\mathrm{CP}} = \frac{\hbar}{\pi}\int_0^{\infty} d\xi \mathrm{Tr}\{\underline{\alpha}(i\xi) \cdot \partial_z \underline{G}(\mathbf{r}_a, \mathbf{r}_a, i\xi)\}}$$

Eg.: Harmonic oscillator model

- Exact polarizability $$\underline{\alpha}(\omega) = (e^2/m)[\omega_a^2 - \omega^2 - (e^2/m)\underline{G}(\mathbf{r}_a, \mathbf{r}_a, \omega)]^{-1}$$

- Green tensor: vacuum+scattering contributions $$\underline{G} = \underline{G}_v + \underline{G}_s$$

- Vacuum-dressed polarizability $$\underline{\alpha}_0(\omega) = (e^2/m)[\omega_a^2 - \omega^2 - (e^2/m)\underline{G}_v(\omega)]^{-1}$$

$$\underline{\alpha}(\omega) = [1 - \underline{\alpha}_0(\omega) \cdot \underline{G}_v(\omega)]^{-1}$$

- Standard scattering formula

$$F_{\mathrm{CP}} = -\frac{\hbar}{2\pi}\frac{\partial}{\partial_z}\int_0^\infty d\xi \mathrm{Tr}\log[1 - \alpha_0(i\xi) \cdot \underline{G}_s(\mathbf{r}_a, \mathbf{r}_a, i\xi)]$$

CP: Quantum regression

- Onsager regression theorem: *The average regression of fluctuations obeys the same laws as the corresponding irreversible process* (Onsager 1931)

- Quantum regression hypothesis (aka "theorem", QRT) (Lax 1963)

$$\underline{C}(t, t-\tau) \equiv \langle \mathbf{d}(t)\mathbf{d}(t-\tau)\rangle = \langle \mathbf{d}^2(t)\rangle e^{-i(\omega_a - i\gamma_a/2)\tau}$$

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- Widely used in quantum optics
- Approximate: weak system-bath coupling, near resonance (Ford+O'Connell 1996)
- Exact quantum generalization of Onsager regression: FDT
- FDT and QRT predict different decay of correlations

  - "Short" times ($\tau\gamma_a \ll 1$): exponential decay $\mathrm{QRT} = \mathrm{FDT}$
  - "Large" times ($\tau\gamma_a \gg 1$): power-law decay $\mathrm{QRT} \neq \mathrm{FDT}$

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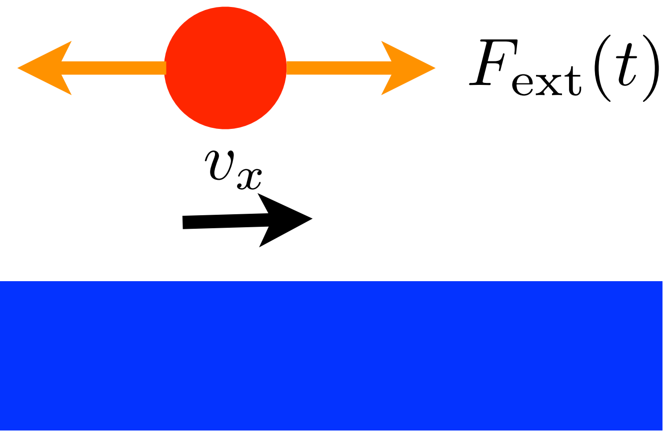
- Stationary CP force using QRT

$$F_{\mathrm{CP}} = \frac{\hbar}{\pi}\int_0^\infty d\xi \mathrm{Tr}\left\{\frac{\underline{\tilde{\alpha}}(i\xi) + \underline{\tilde{\alpha}}(-i\xi)}{2}\cdot \partial_z \underline{G}\right\}$$

$$\underline{\tilde{\alpha}}(i\xi) = (\mathbf{dd}/\hbar)[(\omega_a^2 + i\xi - i\gamma_a/2)^{-1} + (\omega_a^2 + i\xi + i\gamma_a/2)^{-1}]$$

Non-equilibrium case

$$F_x(t) = \langle \hat{\mathbf{d}}(t) \cdot \partial_x \hat{\mathbf{E}}(\mathbf{r}_a(t), t) \rangle$$



- Ground state atom

- Prescribed motion

$$\mathbf{r}_a(t) = \begin{cases} (x_a, y_a, z_a) \text{ for } t \leq 0 \\ (x_a + v_x t, y_a, z_a) \text{ for } t > t_s \end{cases}$$

$$m_a \ddot{x}_a(t) = F_{\text{ext}}(t) + F_x(t)$$

- Stationary $(t \to \infty)$ frictional force

$$F_{\text{fric}} = \text{Re}\left\{ \frac{2}{\pi} \int \frac{d^2\mathbf{k}}{(2\pi)^2} k_x \int_0^\infty d\omega \int_0^\infty d\tau e^{-i(\omega - k_x v_x)\tau} \text{Tr}[\underline{C}(\tau; v_x) \cdot \underline{G}_I(\mathbf{k}, z_a, \omega)] \right\}$$

$$\underline{C}_{ij}(\tau; v_x) = \text{tr}\{\hat{d}_i(0) \hat{d}_j(-\tau) \hat{\rho}(\infty)\}$$

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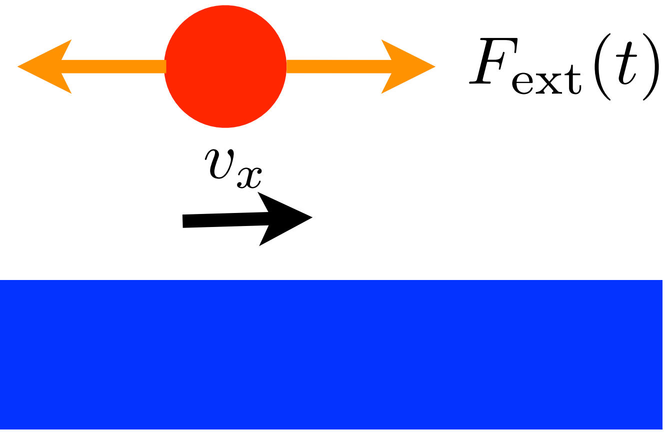

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$$\underline{C}_{ij}(\tau; v_x) = \mathrm{tr}\{\hat{d}_i(0)\hat{d}_j(-\tau)\hat{\rho}(\infty)\} \longrightarrow \hat{\rho}(\infty) = ???$$

NEQ FT and quantum friction

- No general results as in the equilibrium case

However, it is still possible to draw general conclusions about the frictional force in the low-velocity limit.

Chetrite et al. 2008
Baiesi et al. 2009

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- Non-equilibrium power spectrum $\underline{S}(\omega; v_x) = (2\pi)^{-1} \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \underline{C}(\tau; v_x)$

$$\boxed{F_{\text{fric}} = 2 \int \frac{d^2\mathbf{k}}{(2\pi)^2} k_x \int_0^\infty d\omega \, \mathrm{Tr}[\underline{S}_R(k_x v_x - \omega; v_x) \cdot \underline{G}_I(\mathbf{k}, z_a, \omega)]}$$

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- Small velocity analysis: no linear-in-v terms
  - Contributions from $\underline{S}_R(-\omega; v_x)$ cancel upon integration over k_x
  - Contributions from $\underline{S}_R(k_x v_x - \omega; 0)$ $\longrightarrow$ equilibrium FDT!

$$\underline{S}_R(k_x v_x - \omega; 0) = (\hbar/\pi)\theta(k_x v_x - \omega)\underline{\alpha}_I(k_x v_x - \omega)$$

$$\boxed{F_{\text{fric}} \approx \frac{2\hbar v_x^3}{3(2\pi)^2} \int_{-\infty}^{\infty} dk_y \int_0^{\infty} dk_x k_x^4 \, \mathrm{Tr}[\alpha_I'(0) \cdot G_I'(\mathbf{k}, z_a, 0)]}$$

FTD vs QRT and q. friction

- The *exact* FDT predicts a stationary frictional force at zero temperature that scales (at least) as velocity cubed, *independent of the model for the atom's polarizability.*

FTD vs QRT and q. friction

- The *exact* FDT predicts a stationary frictional force at zero temperature that scales (at least) as velocity cubed, *independent of the model for the atom's polarizability.*

- In contrast, QRT gives a linear-in-velocity stationary frictional force

Using the QRT for the correlator in the static case, $\underline{C}(t, t-\tau) = \langle \mathbf{d}^2(t) \rangle e^{-i(\omega_a - i\gamma_a/2)\tau}$

$$F^{\mathrm{QRT}}_{\mathrm{fric}} \approx v_x \frac{d^2\gamma_a}{3\pi} \int \frac{d^2\mathbf{k}}{(2\pi)^2} k_x^2 \int_0^\infty \frac{\omega + \omega_a}{[(\omega+\omega_a)^2 + \gamma_a^2/4]^2} \mathrm{Tr}[\underline{G}_I(\mathbf{k}, z_a, \omega)]$$

- However, in the weak coupling limit $(\gamma_a \to 0)$

$$\mathrm{QRT} = \mathrm{FDT}$$

$$F_{\mathrm{fric}} \propto \exp(-1/v_x)$$

Moving harmonic oscillator

- Model the atom as a linear harmonic oscillator
- Dipole moment $\hat{\mathbf{d}} = e\hat{\mathbf{x}}$
- Equation of motion can be solved exactly, including transients

$$\ddot{\hat{\mathbf{d}}}(t) + \omega_a^2 \hat{\mathbf{d}}(t) = (e^2/m)\hat{\mathbf{E}}(\mathbf{r}_a(t), t)$$

- Laplace transform $(t > t_s)$

$$\left[s^2 + \omega_a^2 - \frac{e^2}{m} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \int_0^\infty d\omega \frac{G_s(\mathbf{k}, z_a, \omega)}{s - i(\omega + k_x v_x)} \right] \hat{\mathbf{d}}(s) = \frac{e^2}{m} \hat{\mathbf{E}}_0(s) + s\hat{\mathbf{d}}(t_s) + \dot{\hat{\mathbf{d}}}(\mathbf{t_s})$$

$\hat{\mathbf{E}} = \hat{\mathbf{E}}_0 + \hat{\mathbf{E}}_s$ Free + source field

$$\hat{\mathbf{d}}(t) = \hat{\mathbf{d}}_H(t) + \hat{\mathbf{d}}_P(t)$$

Depends on initial conditions. Decays to zero at large times

Non-equilibrium FDT

- Using the exact solution for the oscillator model, one can prove the following exact, non-equilibrium fluctuation-dissipation relation

$$\boxed{\underline{S}(\omega; v_x) = \frac{\hbar}{\pi}\theta(\omega)\alpha_I(\omega; v_x) - \frac{\hbar}{\pi}\underline{J}(\omega; v_x)}$$

- $\alpha(\omega; v_x)$ dynamic polarizability of moving oscillator

$$\alpha(\omega; v_x) = \frac{e^2}{m}\left[\omega_a^2 - \omega^2 - \frac{e^2}{m}\int \frac{d^2\mathbf{k}}{(2\pi)^2}\underline{G}(\mathbf{k}, z_a, \omega + k_x v_x)\right]^{-1}$$

- Current $\underline{J}(\omega; v_x)$

$$\underline{J}(\omega; v_x) = \int \frac{d^2\mathbf{k}}{(2\pi)^2}\left[\theta(\omega) - \theta(\omega + k_x v_x)\right] \underline{\alpha}(\omega; v_x) \cdot \underline{G}_I(\mathbf{k}, z_a, \omega + k_x v_x) \cdot \underline{\alpha}^\dagger(\omega; -v_x)$$

- Non-equilibrium FDT in classical models have the same form

Chetrite et al. 2008

- Using the $\underline{S}(\omega; v_x)$ above one can verify that $F_{\text{fric}} \propto v_x^3$

Moving two-state atom

- Model the atom as a two-level system (generalization to multi-level possible)
- Dipole moment $\hat{\mathbf{d}} = \mathbf{d}\hat{\sigma}_x$
- Non-linear equation of motion. Exact solution not possible

$$\ddot{\hat{\sigma}}_x(t) + \omega_a^2 \hat{\sigma}_x(t) = -\frac{2\omega_a}{\hbar}\hat{\sigma}_z(t)\,\mathbf{d}\cdot\hat{\mathbf{E}}(\mathbf{r}_a(t),t) \qquad \langle \hat{\sigma}_x(t)\hat{\sigma}_x(t')\rangle = ?$$

- Perturbative solution in powers of the atom-field coupling $\mathbf{d}$

- $\mathcal{O}(\mathbf{d}^2)$: $\underline{C}(t,t';v_x) \approx \mathbf{d}\mathbf{d}e^{-i\omega_a(t-t')} \longrightarrow F_{\text{fric}} \propto \exp(-1/v_x)$
- $\mathcal{O}(\mathbf{d}^4)$:

$$\ddot{\hat{\sigma}}_x(t) + \omega_a^2(t)\hat{\sigma}_x(t) + \frac{2}{\hbar^2}\int_0^t dt_1\{\mathbf{d}\cdot\hat{\mathbf{E}}_0(\mathbf{r}_a(t),t),\mathbf{d}\cdot\hat{\mathbf{E}}_0(\mathbf{r}_a(t_1),t_1)\}\dot{\hat{\sigma}}_x(t_1) = -\frac{2\omega_a}{\hbar}\hat{\sigma}_z(0)\mathbf{d}\cdot\hat{\mathbf{E}}_0(\mathbf{r}_a(t),t)$$

$$\longrightarrow \underline{S}(\omega;v_x) \approx \frac{4\omega_a^2}{\pi\hbar}\mathbf{d}\mathbf{d}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\frac{\theta(\omega+k_xv_x)\mathrm{Tr}[\mathbf{d}\mathbf{d}\underline{G}_I(\mathbf{k},z_a,\omega+k_xv_x)]}{[\omega_a^2-(\omega-i0^+)^2][\omega_a^2-(\omega+i0^+)^2]}$$

$$\longrightarrow F_{\text{fric}} \propto v_x^3$$

Conclusions

- Atom-surface quantum friction from general non-equilibrium stat. mech.
- $\mathrm{QRT} \neq \mathrm{FDT}$
- Non-equilibrium FDT predicts a cubic-in-v frictional force
- At high temperatures (classical limit), $\mathrm{QRT} = \mathrm{FDT}$, and linear-in-v friction
- Same analysis possible for quantum friction between macroscopic bodies

- Note: all the above is valid in the *true stationary, long-time limit*, after all transients have died out. *For shorter times*, the atom-friction force is linear-in-v, in agreement with (some) previous calculations